THE MATHEMATICAL SOLUTION TO INTERLOCK IN CONCRETE BLOCK PAVING

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Introduction

Several guides have been produced[1,2,3] which provide information allowing the designer to proportion and specify the road foundation and structure for concrete block roads. Essentially, materials of sufficient strength and thickness need to be provided such that the underlying subgrade will neither deform nor deflect excessively. Figure 1 shows the three components present in all pavements; the surface, the structure and the foundation. The purpose of the surface is to provide a durable, non skid, non slip, rideable and attractive upper surface to the pavement. In the case of concrete block paving, in addition to fulfilling these requirements, observation and research has shown that a significant structural benefit accrues such that the underlying structure can be reduced in strength compared with that which would be needed for a zero strength surfacing material e.g. tar spray surface dressing.

The structural benefit provided by concrete block pavers has been defined in term of *interlock* and Knapton[4] originally defined interlock as the inability of an individual paver to move independently of its neighbours. He categorised interlock into its three components viz.: rotational, vertical and horizontal. Figure 2 illustrates how rotational interlock is achieved by providing edge restraint and how vertical interlock is provided by the joint filling material. Horizontal interlock is achieved by herringbone laying pattern. A pavement incorporating full interlock can sustain high levels of applied load. An interlocking paver pavement can be observed to behave in a manner whereby the pavers act together as a system such that the pavement has more of the character of a flexible homogeneous material rather than that of a collection of individual units behaving independently. Because of this, virtually all of the research into the behaviour of concrete block paving has focused upon establishing relative performance factors between pavers and conventional flexible pavement construction materials

The principal UK paver pavement design guide, BS7533:1992[5] uses conventional flexible pavement design technology with a one to one equivalence factor between pavers and asphalt, i.e. 100mm thickness of bedding sand and pavers is equivalent structurally with 100mm asphalt. This assumption is based upon previous research[6,7] which is supported by observations of the performance of pavers on trafficked areas[8] . Because the equivalence thickness technique has been found to be adequate in day to day design, there has been no need to attempt to investigate rigorously the mechanics of the behaviour of concrete block paving. Woodman[9] has reported full scale testing in which levels of vertical strain were measured beneath rolling loads at the Transport Research Laboratory(TRL). He has concluded that pavers and their bedding sand equivalence with no more than 10mm asphalt, rather than the value of 100mm in BS7533. Other

unpublished experimental work at TRL has indicated that concrete pavers can actually magnify applied pressure such that stresses measured in the bedding sand have been found to exceed the tyre pressure of the load vehicle.

There is clearly a contradiction between the findings of some researchers and the assumptions made by designers and by those drafting codes of practice. The apparently unconservative design assumptions being made world-wide on a daily basis are not in fact leading to the premature pavement deterioration levels which some research suggests should be occurring. This paper seeks to resolve this contradiction by examining in detail the way in which patch loading applied to the wearing surface of pavers is transmitted as vertical stress into the underlying bedding material. It develops the equations of vertical and rotational equilibrium which are solved to determine the values of critical stresses beneath pavers. It first of all considers the case of zero interlock i.e. each paver comprises an individual unit unconnected to its neighbours and shows that the mathematical solution to this case results in stresses which exceed applied pressure by 70%. This is important because it means that any attempts to measure the effect of interlock should in fact compare measured stresses not with tyre pressure but with a value 70% greater than tyre pressure. The paper concludes that those researchers who have made an assessment of the equivalence of pavers with asphalt on the basis of comparing measured stress with actual applied pressure have drawn inaccurate conclusions. It therefore explains the contradiction between the practitioners and the researchers and explains why the design assumption of one to one equivalence remains valid.

The paper goes on to illustrate steps in the patterns of vertical stress beneath non-interlocking neighbouring pavers. It explains that those steps can occur only in the theoretical non-interlocking case and uses the values of the stress steps to evaluate (for the first ever time) the true values of interlock which must be established beneath neighbouring pavers. The importance in this evaluation of interlock stress is in the fact that it can be used to explain many phenomena known to exist in pavers which so far have been considered to be too difficult to understand. For example, it is shown that when joint widths exceed 5mm, the interlock potential of pavers diminishes rapidly and this finding agrees with the observed performance of pavers. The paper also compares different laying pattern orientations and confirms the commonly held, but until now unsubstantiated view that 45 degree herringbone pattern is the preferred laying arrangement.

Patch Loading of Pavers

Figure 3 illustrates the plan relationship between the contact zone of a commercial, vehicle's dual rear wheels and pavers laid at an angle to the direction of the vehicle. This is a common laying arrangement on highways and on industrial areas. The shaded areas represent full contact between tyre and pavers and have been produced by applying wet paint to a vehicle's tyres and lowering it onto pavers. The tyres were inflated to a pressure of $0.8N/mm^2$ and a load of 10,000kg was applied through the entire axle such that each individual wheel transmitted 2,500kg into the pavers. Figure 3 shows that at no time is one paver loaded over its entire plan area. This means that loading is applied asymmetrically to each paver at all times and the resulting pattern of compressive stress in the underlying bedding sand cannot be calculated on a simple load spreading basis. Before the stresses in the bedding sand can be analysed, it is necessary to select a particular patch loading on a paver.

Figure 4 shows representative examples of patch loading arrangements on pavers from which it can be seen that in the general case the centre of gravity of the paver is displaced from the centre of applied pressure. In the general case, the two centres are displaced in both the transverse and the longitudinal directions of the paver. The distances a, b, c and d characterise the patch loading and are used in the development of bedding sand contact pressure. In those calculations, a paver of thickness 80mm and plan dimensions 200mm x 100mm with a chamfer of width and depth 5mm around its upper perimeter is used.

Bedding Sand Stress Calculation Method

The vertical stresses in the bedding sand are calculated by considering vertical and rotational equilibrium of the pressures applied to the upper and lower horizontal paver surfaces. It is assumed that the paver is structurally rigid in relation to the bedding sand so a planar stress regime can be assumed at the paver/sand interface. This assumption is justified by the values of Young's Modulus which would normally be applied to concrete ($30{,}000N/mm^2$) and to sand in compression ($300N/mm^2$). If tension were to be permitted to develop between the paver and the bedding sand the solution would be relatively straightforward. Tension cannot be developed in uncemented sand so it is assumed that compressive stress is developed in some parts of the interface and zero stress is developed elsewhere.

By ensuring vertical and rotational equilibrium, three equations can be developed (one vertical equilibrium and two rotational equilibrium equations, one in each orthogonal direction) which can be solved to obtain the values of the vertical stress in the bedding sand at each corner of the lower horizontal surface of the paver. The stress at any point along the paver boundary can then be determined by linear interpolation.

Figure 5 shows the five principal bedding sand stress patterns. The five cases can be best understood by considering them sequentially. Initially, the applied upper surface patch load occupies one corner of the loaded paver. Bedding sand compressive stress occurs only at one lower paver corner and zero stress occurs at the remaining three corners. As the patch load occupies progressively more paver surface area, the situation illustrated is attained in which stresses are compressive at two paver corners. Eventually, the patch load creeps further over the paver so that three corners exhibit compressive bedding sand stress. Finally, compressive stress develops at each of the four corners. No matter how complex the patch loading might be, one of the five conditions in Figure 5 must represent the true state of vertical stress at the paver/bedding sand interface.

The evaluation of paver corner stresses can be understood by reference to the first of the stress blocks in Figure 5. For rotational equilibrium, the stress block centroid co-ordinates must be equal to the easily calculable patch load centroid co-ordinates, hence the determination of values for the two stress block lengths **l** & **t**. Once those two lengths have been so determined, vertical equilibrium can be used to determine the volume of the stress block **V** (which has a dimension of force) since its value must equal the total downward force applied to the paver. Once **V**, **l** and **t** are known, it is a simple matter to evaluate σ_{Az}.

Similar reasoning can be applied to the second and third stress blocks in Figure 5 in which case values are required for σ_{Az} and σ_{Bz}. In these cases, there are three unknowns, σ_{Az} , σ_{Bz} and **f**, the vertical stress run-out distance from the origin along the paver sides. The three non-linear equations to be solved are the equation of vertical equilibrium and

the two orthogonal equations of rotational equilibrium. The solution when the two corner stresses are separated by a long side is:

$$\sigma_{A_z} = \frac{N(100-y+a)(-20,000+3y^2+400a-3ya)}{12,000,000x(y-50)}$$

$$\sigma_{B_z} = \frac{N(-20,000+3y^2+400a-3ya)}{6,000,000x}$$

$$f = \frac{3600x(50-y)(100-y+a)}{(-100+3y+a)(20,000-3y^2-400a+3ya)}$$

Where:

$a = \sqrt{-20,000+600y-3y^2}$
$N = \sum Load$
$x =$*Applied load short side centroidal distance*
$y =$*Applied load long side centroidal distance*

Note that for this solution to be valid, y must lie in the range: $42.265 \le y \le 157.735mm$

When the compressive stresses are separated by a shorter rather than by a longer side, t is the vertical stress run out distance along the paver longer side. The corner stresses are:

$$\sigma_{A_z} = \frac{N\left[(50-x+b)\left(-5,000+3x^2+200b-3xb\right)\right]}{1,500,000y(x-25)}$$

$$\sigma_{C_z} = \frac{N\left(-5000+3x^2+200b-3xb\right)}{750,000y}$$

$$t = \frac{1800y(25-x)(50-x+b)}{(-50+3x+b)\left(5000-3x^2-2000b+3xb\right)}$$

Where:

$b = \sqrt{-5,000+3x-3x^2}$
$N = \sum Load$
$x =$*Applied load short side centroidal distance*
$y =$*Applied load long side centroidal distance*

Note that for this solution to be valid, x must lie in the range: $21.1325 \le x \le 78.8675mm$

Figure 5 shows the stress pattern when compressive stress occurs at three paver corners. Rotational equilibrium requires that the centroid of the stress block corresponds with the centroid of the load patch, hence values for the stress block centroid co-ordinates. The solution to these equations is particularly complex so an iterative solution has been used in their evaluation.

The final part of Figure 5 shows compressive stress at each of the four paver corners. The solution follows the pattern of the solutions above such that three non-linear equations are established in σ_{Az}, σ_{Bz} and σ_{Cz}. The solution is as follows:

$$\sigma_{Az} = \frac{N(700 - 6x - 3y)}{2{,}000{,}000}$$

$$\sigma_{Bz} = \frac{N(100 - 6x + 3y)}{2{,}000{,}000}$$

$$\sigma_{Cz} = \frac{N(100 + 6x - 3y)}{2{,}000{,}000}$$

Where:

$N = \sum Load$
x = *Applied load short side centroidal distance*
y = *Applied load long side centroidal distance*

Once those three corner stress values have been determined, the fourth corner stress σ_{D_z} can be determined from the following equation which is true as a result of the planarity of the stress variation:

$$\sigma_{D_z} = \sigma_{C_z} + \sigma_{B_z} - \sigma_{A_z}$$

Evaluation of Stresses in Bedding Sand with & without Interlock

Figure 6 shows the development of paver corner stresses in the case of a chamfered rectangular paver of dimensions 100mm x 200mm. The maximum stress beneath a 0.8N/mm^2 patch load is 1.15N/mm^2 i.e.45% overstress when only one corner of the paver develops positive stress. Figures 7, 8 and 9 show the pattern of stresses which develop within the bedding sand when a vehicle passes over rectangular pavers laid to a herringbone pattern. Figure 7 applies when the vehicle runs at angles of zero, 10° and 20°(as defined in Figure 3) to the orientation of the laying pattern, Figure 8 applies when the vehicle travels at 30°, 40° and 60° and Figure 9 applies when the vehicle travels at 70°, 80° and 90°.

Each of Figures 7,8 and 9 shows bedding sand stresses both with and in the absence of interlock. The non-interlocking stresses are taken from the solutions to the equations set out in the last Section. In the case of interlock, the definition of interlock set out in the Introduction is invoked as follows. If neighbouring pavers cannot move independently of one another, the bedding sand beneath them must accept similar compressive stress

values since any difference in neighbouring stresses would lead to differential compaction of the sand and loss of vertical interlock. Therefore, along all of the paver boundaries, the stresses have been modified by taking the average of the two values along the sides of adjoining pavers. This is accomplished by assessing corner stress values and stress runout distances where appropriate.

The stresses in Figures 7, 8 and 9 are shown at points defined in Figure 3. The lines joining the points show variation of stress along paver sides. Effectively, the lines shown in Figures 7, 8 and 9 define the peak stresses in bedding sand and also provide an indication of the effectiveness of interlock for each vehicle/laying pattern orientation. Graphs with high peak stress values and low intermediate values indicate relatively ineffective interlock whereas uniform values with low peaks indicate more effective interlock. One interesting conclusion is that pavers are likely to perform better when their laying pattern runs in line with or at right angles to the direction of traffic (θ=0° or 90°). When vehicles deviate by as little as 10° from these two directions, interlock performs less effectively.

Implication of Results

Figures 3, 7, 8 and 9 show that the true behaviour of concrete block paving under highway loading can be understood only when the way in which asymmetrically loaded pavers transmit wearing surface applied pressures into the underlying bedding sand has been studied. Even when full interlock is developed, any attempt to measure vertical stress in bedding sand will produce results varying from zero to nearly 40% greater than applied pressure according to the position where stresses are measured and according to the orientation of the load patch in relation to the laying pattern. The simplistic concept of spreading applied load through pavers such that stress gradually diminishes with depth does not apply to pavers. For this reason, any attempt to compare the behaviour of pavers with that of homogeneous material by comparing vertical stress in the bedding sand may lead to erroneous results.

Figures 3, 7, 8 and 9 show that pavement analysis must take into account the true shape of the patch loading. Many analytical design procedures assume that the load patch is a disk whose diameter can be determined on the basis of assessing wheel load and contact stress. Such an approach must now be regarded as providing results of limited value in terms of the performance of the pavers and their bedding sand, although may still be valid in terms of overall pavement design. Any assessment of the contribution of different laying patterns or different paver size or shape to overall pavement performance must be based upon the type of analysis presented in this paper.

Figures 3, 7, 8 and 9 also demonstrate why full scale testing of pavers must always be undertaken using true load patch size, pressure and shape and why rolling loads must be used if meaningful results are to be produced. Early research into the behaviour of pavers[4] was based upon measuring stresses in bedding sand when the paver wearing surface was loaded by a static circular plate. Whilst the results provided an initial understanding of the behaviour of pavers and demonstrated the existence of interlock, the inability of such methods to register stress magnification generated at paver corners led to sand specifications being inadequate in some circumstances.

The reason why some bedding sands have performed poorly under a combination of regular traffic and the presence of water can now be understood. A combination of high levels of stress which this paper shows to frequently develop in bedding sands and rapid

changes in those stress values, in conjunction with hydrostatic pressure, are considered by the Authors to have been the cause of several paver pavement failures. Rapidly changing stress values lead to the development of normal stresses in water when the water cannot flow freely, such as is the case in sands including a significant proportion of fine material (defined as finer than 75 microns). In some cases, for example, even when full interlock is operating, bedding sand stress can change from $0.1N/mm^2$ to $0.9N/mm^2$ over a distance of 100mm. A vehicle travelling at 30mph(50kph) would cover 100mm in less than a hundredth of a second. This means that stress can change at a rate exceeding $100N/mm^2$/second. Such a rapid change would lead to destabilising pressures developing in saturated bedding sands. This paper indicates that specifiers should avoid those bedding materials which would be disturbed by a combination of high levels of stress and moisture. In particular, material finer that 75 microns is considered to be likely to lead to bedding sand weakness and should be kept to a minimum.

The complexity of the stress distribution in bedding sand may explain why some research conclusions e.g. Woodman[9] have contradicted observed performance of pavers. The high stress "spikes" shown in Figures 7, 8 and 9 occur for short time periods in a rolling load situation and may be of little consequence when pavers are laid over some materials but may be more important in other situations. For example, it may be the case that pavers seem to perform better when laid over concrete bases than they do when laid over softer bitumen bound materials (better in the sense that even when the additional stiffness inherent in concrete as compared with asphalt is taken into account)

Conclusions

It has been demonstrated that in typical highway loading situations, the pattern of stress development in paver bedding sand is complex and depends upon the size, shape, orientation and speed of the load. Many of the hitherto unexplained bedding sand failures can be understood by examining the way in which local vertical stress peaks develop and travel though bedding sand. In cases where water is present in sands including a significant fraction of fine material, the mechanism whereby sand instability can develop has been explained. Rapidly changing stress levels can result in pressure developing in water in bedding sand which can lead to instability. Many regions permit sand specifications which can lead to instability when vehicles travelling at highway speeds use pavers bedded on saturated sand. It is recommended that such specifications be reviewed and that sands should be specified which permit water to flow as freely as possible where traffic and water may occur concurrently.

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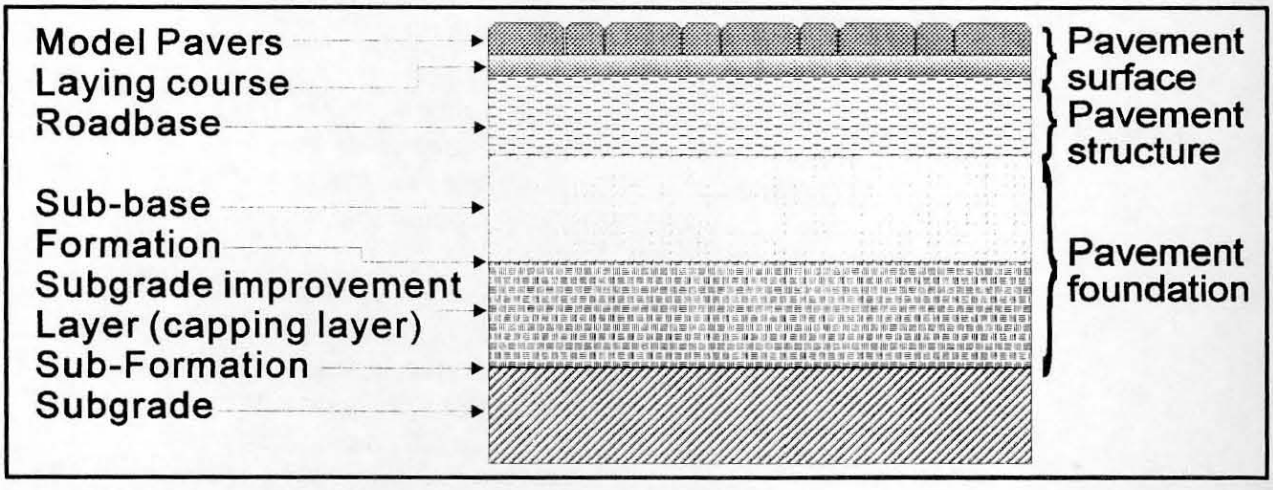


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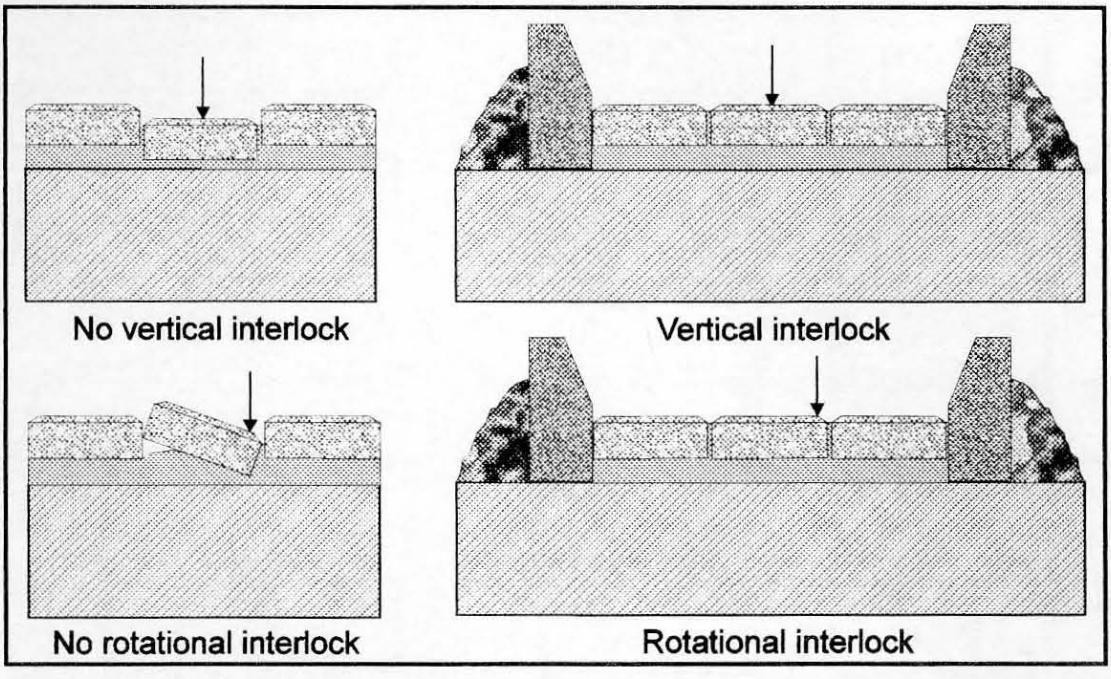


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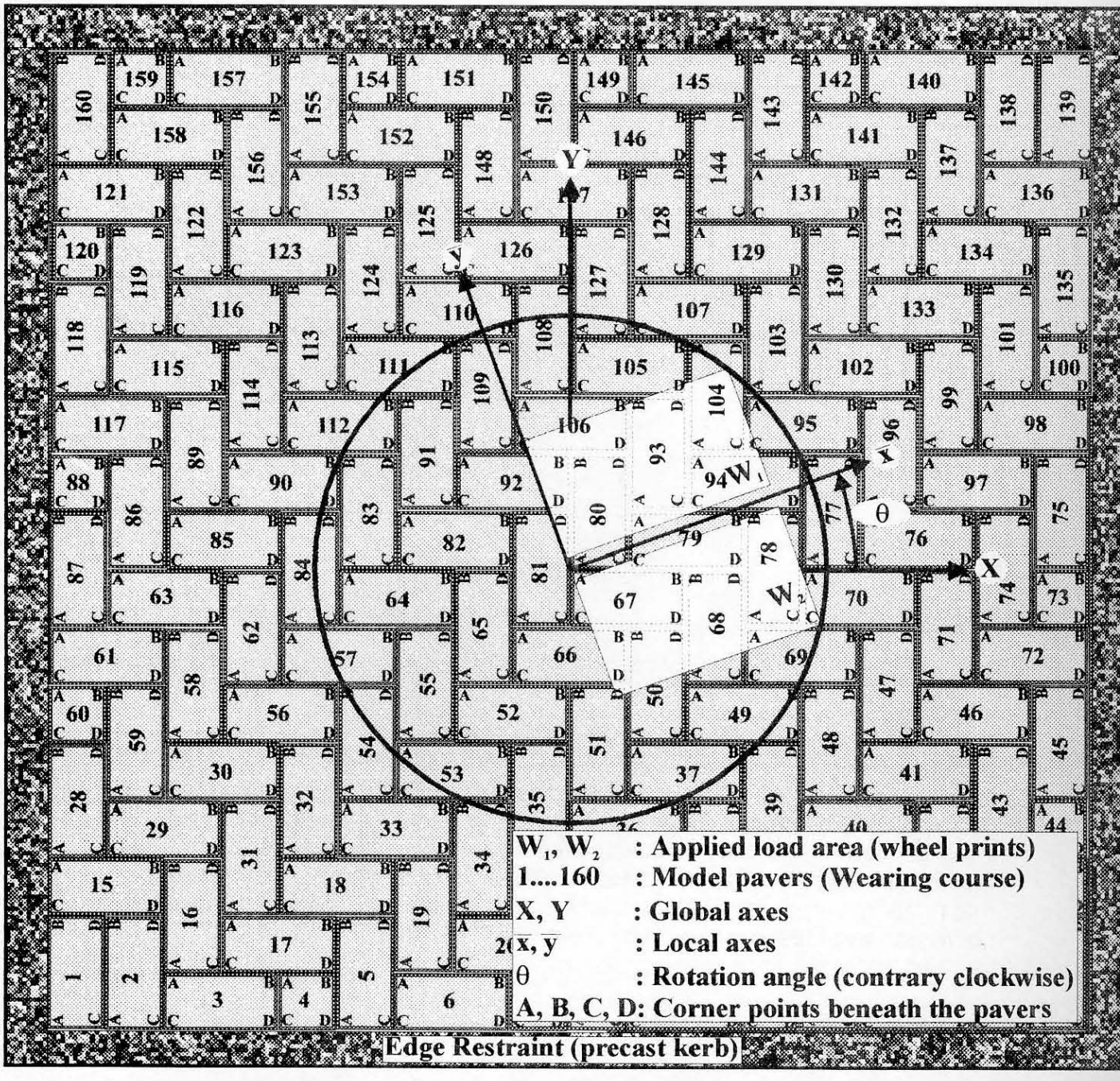


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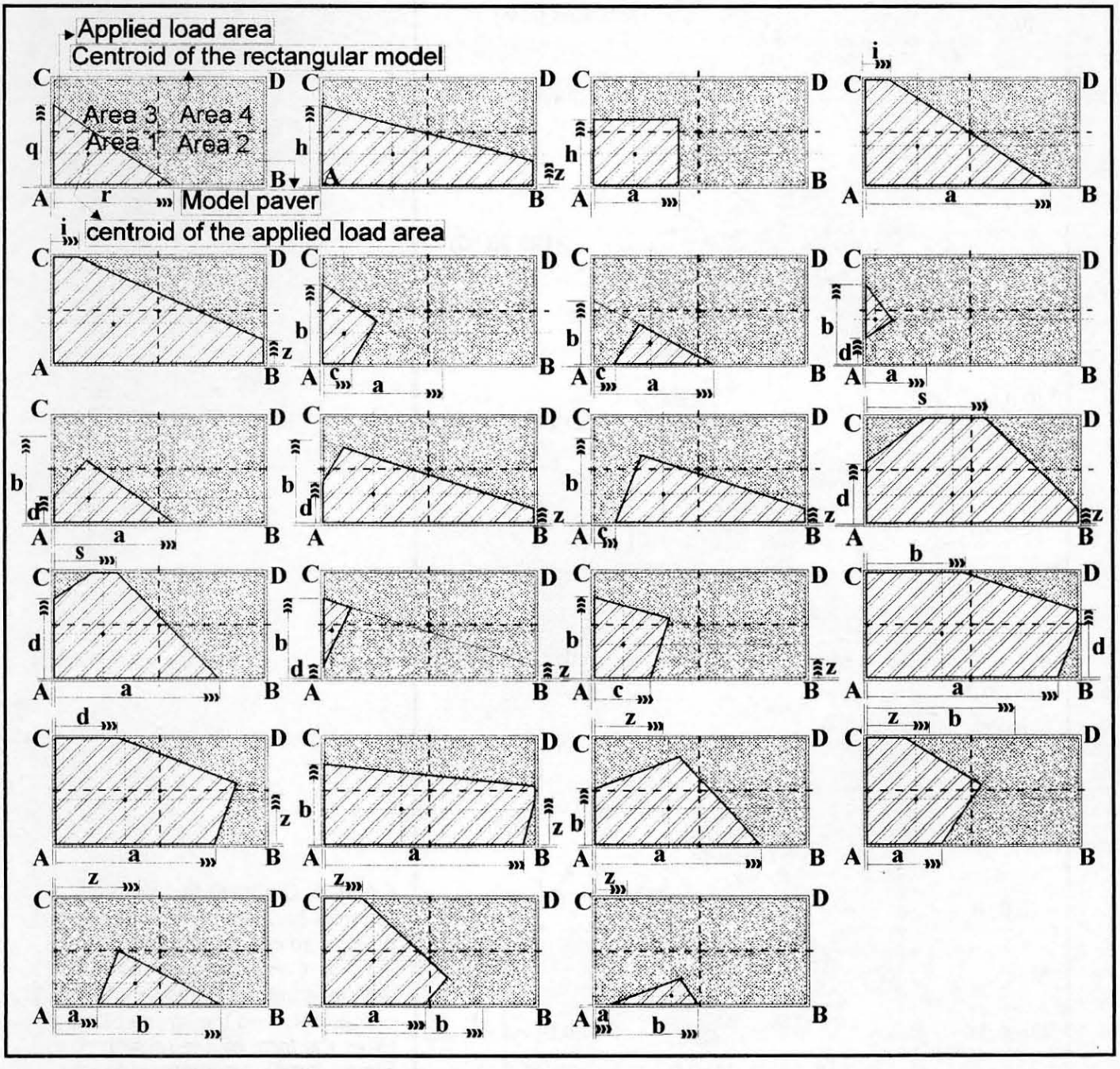


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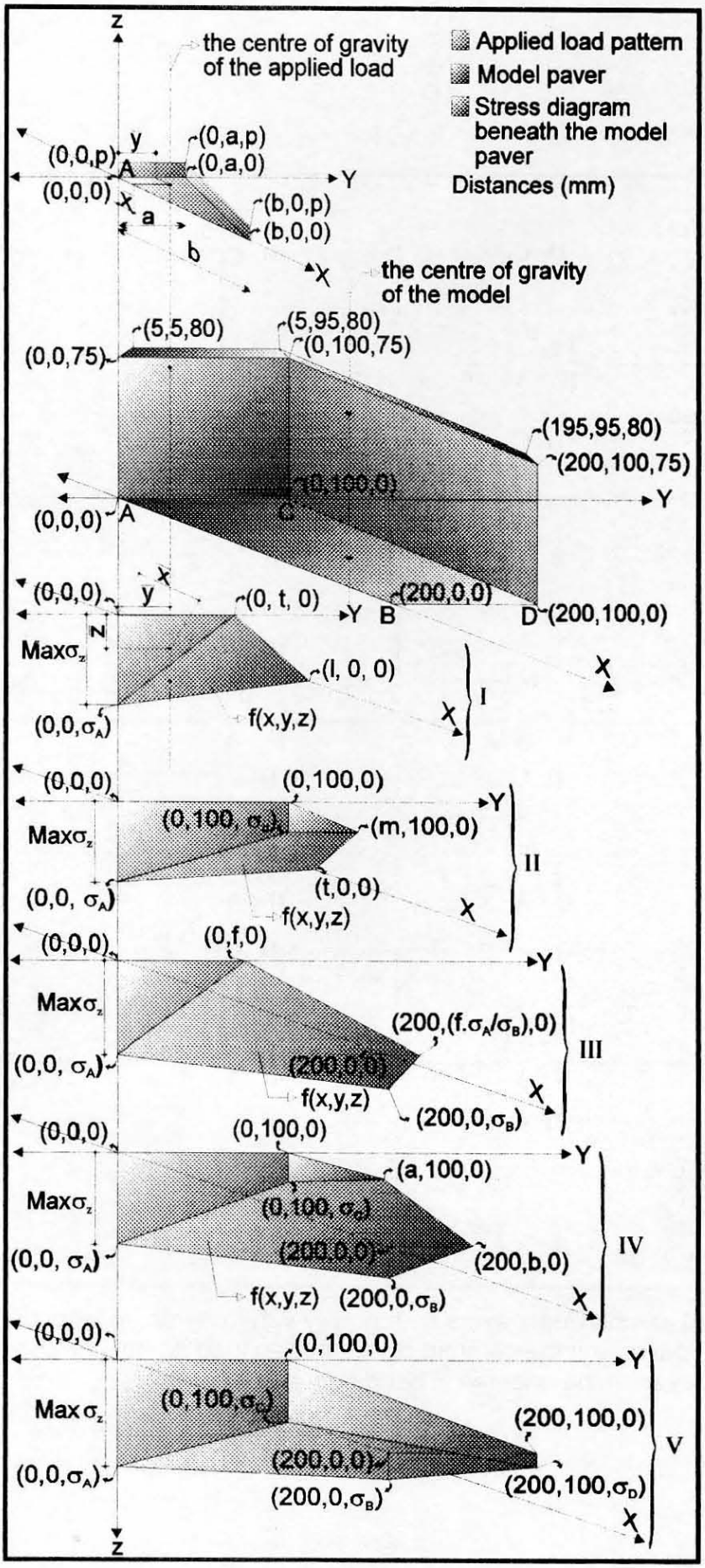


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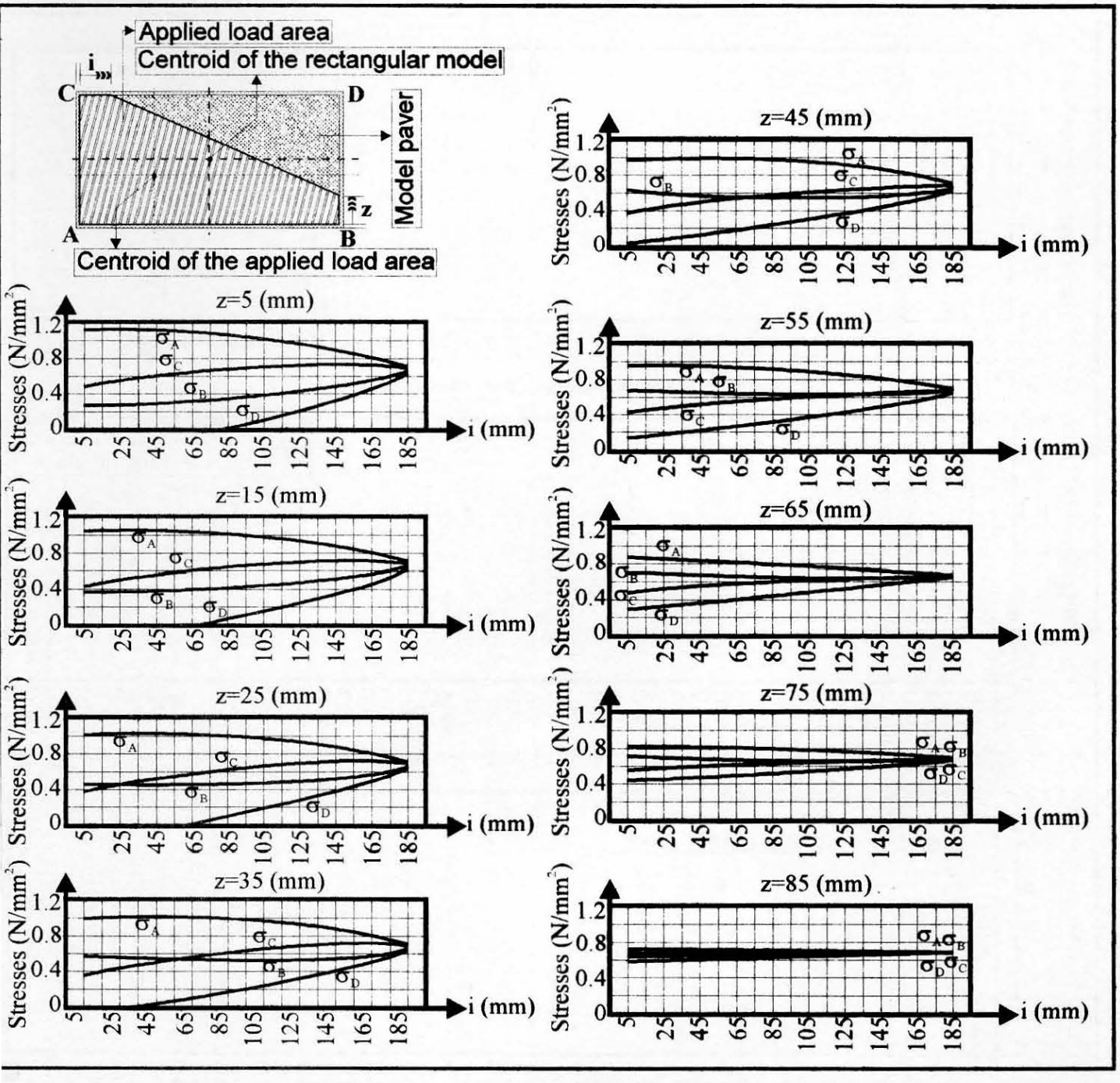


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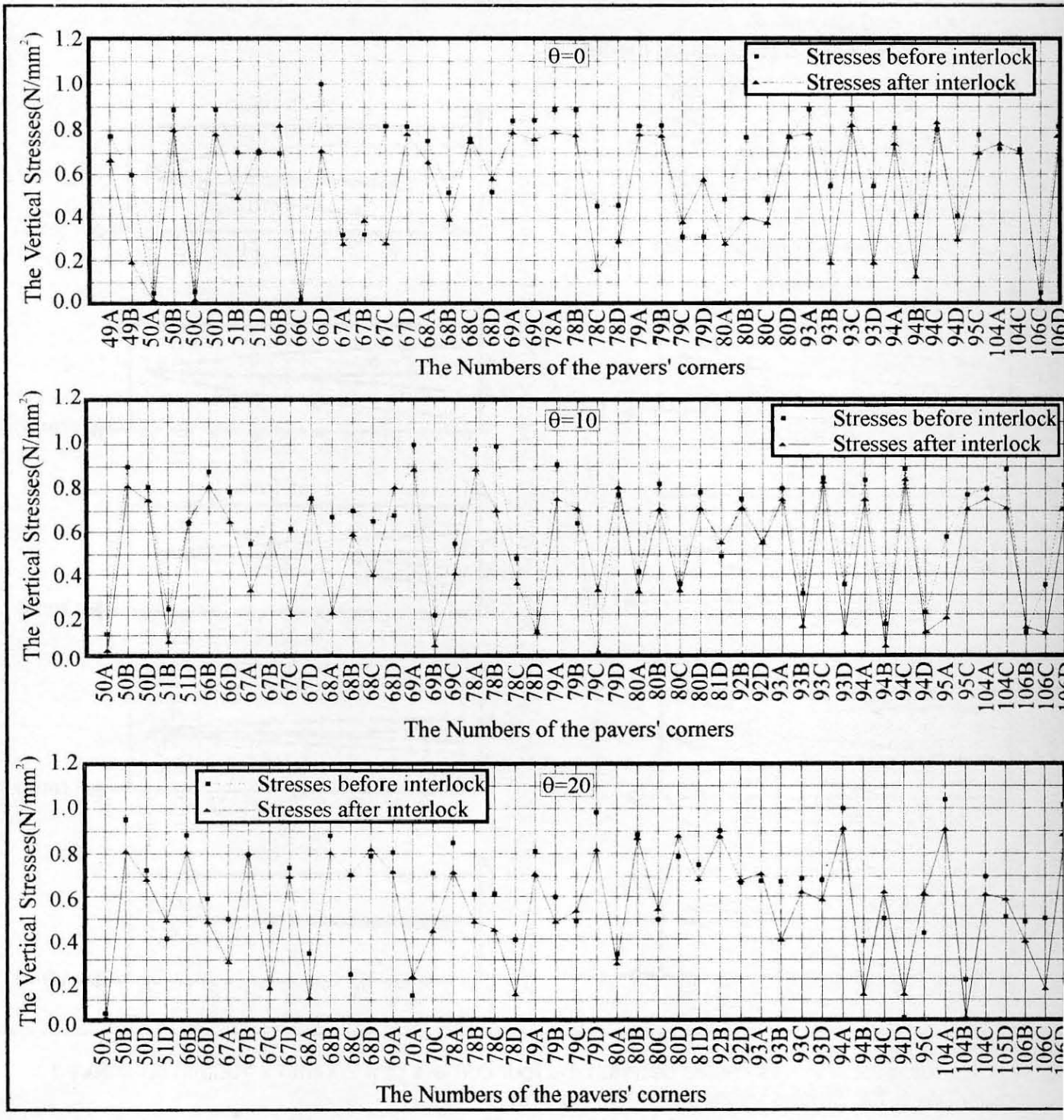


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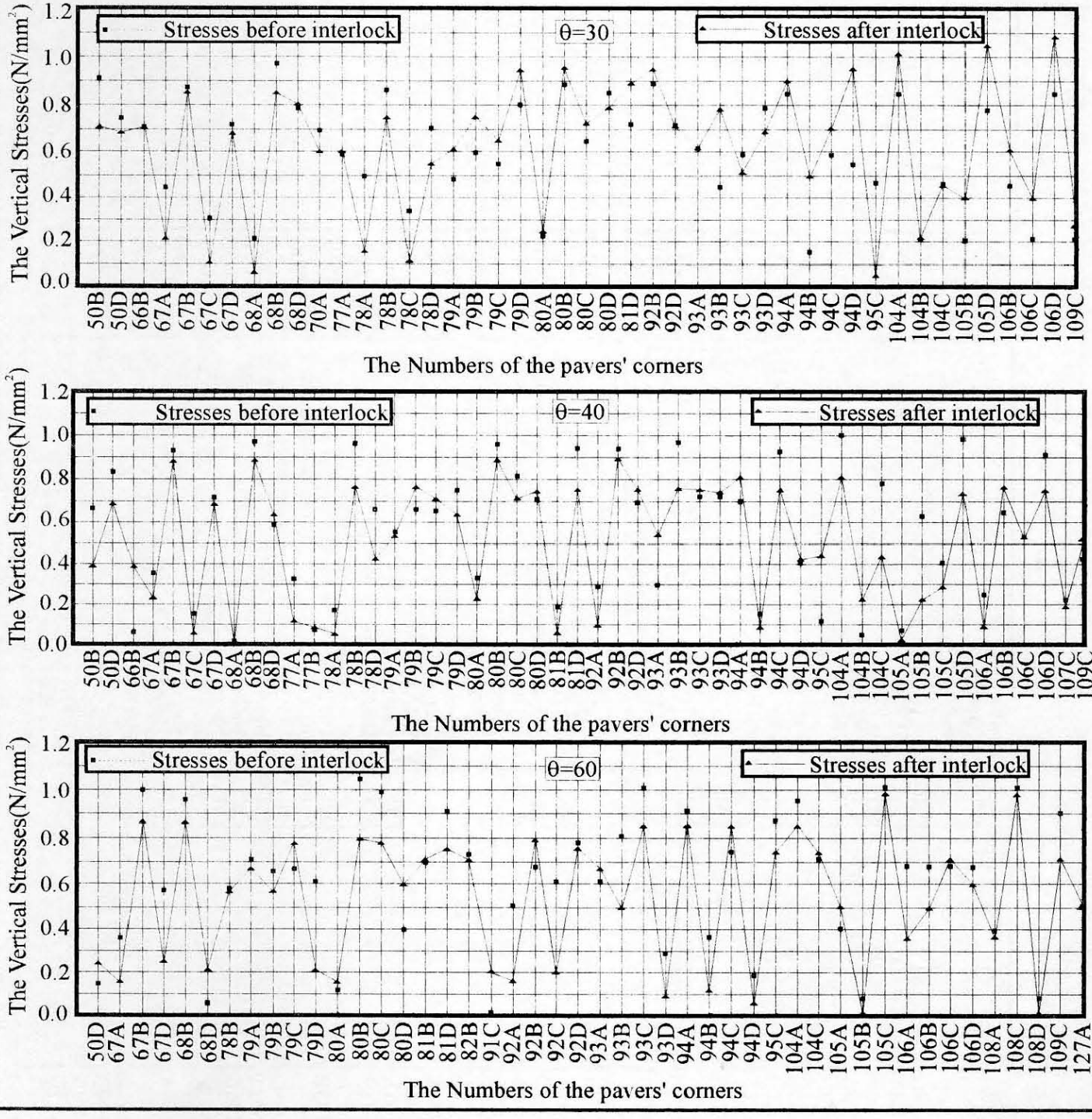


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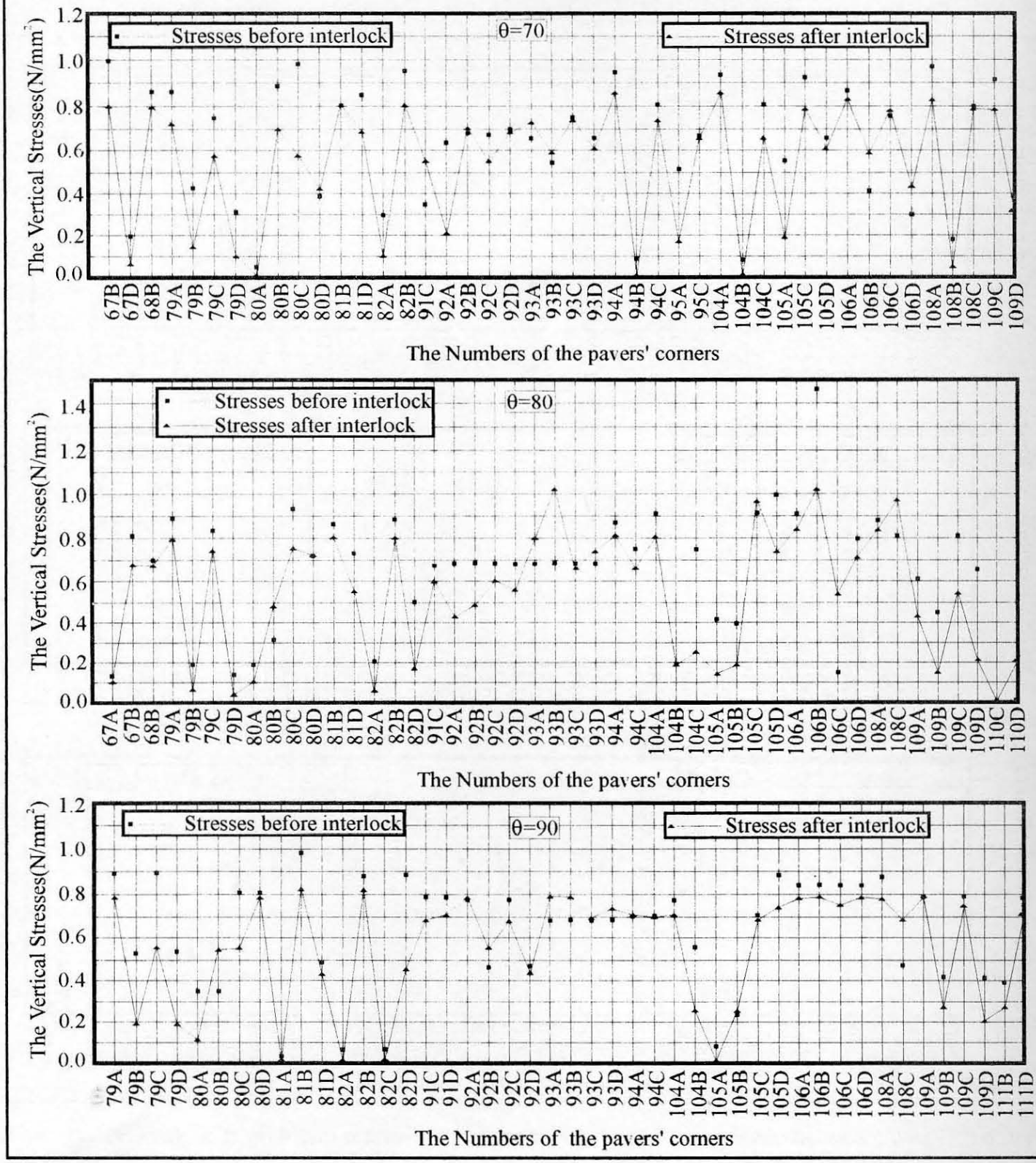


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